Bi-Tetradic Neutrosophy: A Geometric Algebra Formulation

Abstract

We present a four-dimensional geometric (Clifford) algebra formulation of neutrosophy by treating Smarandache's parameters (Truth–Indeterminacy–Falsehood) as vectors and introducing a fourth vector, Reality. Each component is further decomposed into four dialectical vectors (Thesis, Action, Antithesis, Reflection), yielding a bi-quaternionic-like structure with a 16-term summation. Each term contains both scalar and bivector contributions. A brief discussion outlines possible connections to depth-philosophical constructs (ego vs. essence) and dialectical synthesis (quantity vs. quality), as well as potential applications to modelling dynamic synthesis in cognitive and decision systems.

Keywords: neutrosophy; dialectics; non-commutativity; quaternions; synthesis

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1. Introduction

We extend Smarandache's neutrosophic triad (Truth T, Indeterminacy I, Falsehood F) [1] into a four-dimensional geometric (Clifford) algebra by

(1) representing each parameter (T, I, F) as an active vector (analogous to quaternions with zero real part);

(2) introducing a fourth active vector, Reality R;

(3) decomposing each coefficient into four dialectical vectors (Thesis, Action, Antithesis, Reflection); and

(4) constructing a 16-element space with distinct scalar and rotational contributions.

This formalism resembles a bi-quaternionic structure in which the real part is replaced by a fourth imaginary vector. It is motivated by the idea of modelling structured dialectics [2], suggesting that

(1) any proposition can be represented as an ontological vector with multiple aspects,

(2) relations between opposing propositions may be simultaneously contradictory and complementary, naturally leading to a bivector representation, and

(3) each vector layer can be organised into a closed causality loop analogous to the classical four-element schemes, implying a bi-tetradic extension.

2. Preliminaries

We work in the 4-dimensional geometric (Clifford) algebra Cl(4,0) with a Euclidean metric.

- **Neutrosophic basis vectors:** R (Reality), T (Truth), I (Indeterminacy), F (Falsehood).
- **Dialectical basis vectors:** Th (Thesis), Ac (Action), An (Antithesis), Re (Reflection).

Indices i run over {R,T,I,F}, j over {Th,Ac,An,Re}, giving 16 cross-layer combinations.

Operations:

• $\langle x,y\rangle$ = inner (scalar) product.

• $x\wedge y$ = wedge (bivector) product.

• $\star(x\wedge y)$ = Hodge dual mapping bivectors to axial 4-vectors.

3. Bi-Tetradic Coupling

The full coupling is written as:

$$Q = \sum_i \sum_j \left(\alpha_{ij}\langle e_i, u_j\rangle + \beta_{ij} \star \left(e_i \wedge u_j\right)\right) \quad (1)$$

Each term has a **scalar part** $\alpha_{ij}\langle e_i,u_j\rangle$ and a **rotational part** $\beta_{ij}\star(e_i\wedge u_j)$.

Scalar parts capture "amplifying/aligning" effects (additive synthesis), while rotational parts capture "cross-interaction" (emergent synthesis).

Lemma. In 4-D geometric algebra there exist orthogonal vectors e_i,u_j such that $\langle e_i,u_j\rangle=0$ but $\star(e_i\wedge u_j)\neq 0$.

The rotational term genuinely adds new information beyond inner products; it is not reducible to scalar geometry.

4. Basic Properties

• When α dominates over β the system behaves like a linear combination: transitions recombine existing elements without creating novelty.

• When β dominates over α the system produces genuine "dimensional expansion" — new emergent states not reducible to inputs.

Hence, the α/β ratio serves as a **diagnostic of pseudo-synthesis vs. emergent synthesis**.

5. Example

Consider a single cross-layer interaction between **Indeterminacy (I)** from the neutrosophic layer and **Reflection (Re)** from the dialectical layer. Then

$$Q_{I,Re} = \alpha_{I,Re} \langle I, Re \rangle + \beta_{I,Re} \star (I \wedge Re).$$

If $\alpha_{I,Re}$ dominates and $\beta_{I,Re} \approx 0$, the term reduces to a **scalar projection** — a kind of "sophistic reinterpretation" that rearranges ambiguity without changing dimension.

If $\beta_{I,Re}$ dominates and $\alpha_{I,Re} \approx 0$, the term becomes purely **rotational** — a "creative reinterpretation" that yields genuinely new meaning (analogous to poetry rather than sophistry).

(Other cross-layer pairs from the 16-cell grid could be listed in supplementary material.)

6. Discussion: Potential Applications

The dual (scalar + rotational) structure in (1) suggests a natural way of separating alignment from transformation. In a psycho-social interpretation, the scalar term α captures surface-level adaptation (ego/persona or incremental policy), whereas the rotational term β captures dimensional change (individuation or transformative policy shifts). In principle, the ratio α/β could be estimated empirically and used as an index of syntony (constructive resonance) versus anti-syntony (defensive closure) within organisations or societies.

Because the algebra is non-commutative (and, across layers, non-associative), sequences of transformations are path-dependent. This formalism therefore models the irreversibility of experiential learning and the practical difficulty of "unlearning" once a trajectory has been taken.

The two tetrads, (T,R,F,I) and (Th,Ac,An,Re), each form a closed causality loop. Their alignment or misalignment produces patterns that in principle distinguish reality-enhancing from reality-suppressing dynamics. This could provide a quantitative basis for areas that are currently treated only qualitatively or utilitarianly — e.g., assessing whether a policy or practice fosters multi-layer coherence rather than merely increasing a scalar "utility" score.

(*See manuscript version with* *detailed philosophical and psychological interpretations*)

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References

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