

Bi-Tetradic Neutrosophy (*with conceptual interpretations*)

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Abstract

We propose a bi-tetradic neutrosophy framework formulated in four-dimensional geometric (Clifford) algebra. Smarandache's parameters (Truth, Indeterminacy, Falsehood) are treated as vectors and extended by a fourth vector, Reality. Each coefficient is decomposed into four dialectical vectors (Thesis, Action, Antithesis, Reflection), yielding a 16-term structure with separate scalar and bivector contributions. This formalism parallels personality models with both static traits and dynamic transition parameters. The resulting non-commutative algebra offers a new way to represent dynamic synthesis and irreversibility in cognitive and decision systems.

Keywords: neutrosophy; dialectics; non-commutativity; quaternions; VUCA; synthesis

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Introduction

Classical neutrosophic theory establishes the triad (T, I, F) [1], offering considerable philosophical depth but leaving the dynamic synthesis of new, emergent qualities largely implicit. The “Refine/Split” approach subdivides each component into sub-components ($T_1, T_2 \dots, I_1 \dots, F_1 \dots$) [2], increasing granularity but still sharing this limitation. This gap becomes critical in volatile and complex environments, where the order of transformations matters as much as their diversity and magnitude [3, 4].

The challenge lies not only in the number of subcomponents but in their coherent coordination—i.e., which transitions belong together, which follow which, and how to relate them to lived cognitive–affective patterns. We address this by modelling the effects explicitly in four-dimensional geometric (Clifford) algebra and proposing that non-commutativity and the balance of rotational versus scalar dominance correspond to distinct psycho-sociological outcomes.

We treat neutrosophic parameters (T, I, F) as vectors and introduce a fourth vector, R (Reality), which stands in opposition to I (Indeterminacy). This closes the causality loop ($T \leftrightarrow R \leftrightarrow F \leftrightarrow I \leftrightarrow T$) and reveals its non-commutative nature. Each coefficient is then decomposed into four dialectical vectors—Thesis, Action, Antithesis, and Reflection—encoding structured tensions within each component [5]. The resulting sixteen cross-layer combinations correspond to distinct cognitive-affective states with their own synthetic potential.

This yields a bi-quaternion-like algebra that is non-commutative and, across layers, non-associative, formally capturing the irreversible “arrow of experience” and clarifying why some transformation sequences lead to emergent synthesis while others merely recombine existing elements. In this way, the framework provides a basis for identifying and fostering conditions of syntony—states in which strategic orientations and situational processes reinforce one another to generate reality-enhancing (rather than reality-constricting) outcomes.

2. Bi-Tetradic Framework

2.1 Four-Vector Neutrosophic Foundation

We generalize the standard neutrosophic expression into a quaternion-like system:

$$\mathbf{Q} = a\mathbf{R} + b\mathbf{T} + c\mathbf{I} + d\mathbf{F} \quad (1)$$

where the four vectors represent:

- **R (Reality)** — the unknowable ground that cannot be directly accessed or named, corresponding to Korzybski's *territory* [6], Castaneda's *nagual* [7], or Kant's "thing-in-itself" [8]. This serves as active complement to Indeterminacy.
- **T (Truth)** — structured alignment with Reality; its knowable shadow
- **I (Indeterminacy)** — unknown-yet-knowable; the growth frontier
- **F (Falsehood)** — misalignment or distortion of Reality

The critical innovation is treating R as an active vector rather than passive scalar background. This creates two fundamental oppositions: $T \leftrightarrow F$ (knowable alignment vs. misalignment) and $R \leftrightarrow I$ (unknowable ground vs. knowable frontier). These pairs form the basis for dynamic transformations.

2.2 Dialectical Coefficient Structure

Standard neutrosophy cannot generate synthesis because scalar coefficients lack internal tension. We therefore decompose each coefficient as a dialectical vector [5]:

$$\mathbf{q} = \mathbf{eTh} + \mathbf{fAc} + \mathbf{gAn} + \mathbf{hRe} \quad (2)$$

where:

- **Th (Thesis)** — initial proposition or affirmation
- **Ac (Action)** — transformative enactment toward opposition
- **An (Antithesis)** — counter-force capable of negation or complementarity
- **Re (Reflection)** — internalization feeding back into new thesis

This creates structured correspondence: $Th \leftrightarrow T$ (affirmation), $Ac \leftrightarrow R$ (enactment), $An \leftrightarrow F$ (opposition), $Re \leftrightarrow I$ (openness). Each neutrosophic vector now contains internal dialectical dynamics.

Table 1. Comparison of neutrosophic and dialectic parameters

Neutrosophy	Truth (<i>T</i>)	Reality (<i>R</i>)	Falsehood (<i>F</i>)	Indeterminacy (<i>I</i>)
Definition	Structured alignment with Reality, shadow of the bulk	Unknowable ground; the bulk “territory” underlying all maps	Misalignment, distortion of Reality	Unknown-yet-knowable; growth frontier
Stance / Role	Gnosticism, clarity, coherence, evidence	Transcendent / foundational	Distorting / defensive	Ambiguous / fertile
Dialectics	Thesis (<i>Th</i>)	Action (<i>Ac</i>)	Antithesis (<i>An</i>)	Reflection (<i>Re</i>)
Definition	Initial proposition or assertion	Direct enactment transforming Thesis into outcomes	Counter-force opposing or complementing Thesis	Integration of Antithesis back into new Thesis
Stance / Role	Affirmation / starting point	Generative / enactment, activist	Critical / complementary	Integrative / balancing

The neutrosophic layer represents deep, relatively stable vector orientations — the strategic convictions or worldview of a system — while the dialectical layer captures the dynamic processes through which those convictions are enacted. Our goal is to align these two layers across systems and individuals.

2.4 Bi-Tetradic Coupling

Combining equations (1) and (2) produces the full bi-tetradic expression in the four-dimensional geometric (Clifford) algebra terms:

$$Q = \sum_i \sum_j \left(\alpha_{ij} \langle e_i, u_j \rangle + \beta_{ij} \star (e_i \wedge u_j) \right) \quad (3)$$

Summations range over $i \in \{R, T, I, F\}$ and $j \in \{Th, Ac, An, Re\}$, covering all 16 cross-layer combinations. Each combination contributes a scalar term $\alpha_{ij} \langle \mathbf{e}_i, \mathbf{u}_j \rangle$ and a rotational term $\beta_{ij} \star (\mathbf{e}_i \wedge \mathbf{u}_j)$.

Here $\langle \mathbf{e}_i, \mathbf{u}_j \rangle$ is the 4-D inner product (a scalar value, generalizing the “dot product”), $(\mathbf{e}_i \wedge \mathbf{u}_j)$ is a bivector, and \star denotes the Hodge dual, which turns that bivector into a 4-D axial vector (a generalization of the “cross product”).

Lemma. In 4-D geometric algebra there exist orthogonal vectors e_i, u_j such that $\langle e_i, u_j \rangle = 0$ but $\star (e_i \wedge u_j) \neq 0$.

Therefore the rotational layer in (3) represents genuinely new structure that cannot be subsumed under scalar inner products alone.

We propose that this dual (scalar + vector) structure mirrors depth psychology [9]: scalar terms correspond to ego/persona manifestation, while rotational terms capture deeper individuation processes due to dimensional expansion. It can also mirror two types of dialectical synthesis [5]:

- **Pseudo-Synthesis** occurs when scalar terms (α) dominate, producing quantitative combination without qualitative transformation.
- **Emergent Synthesis** occurs when rotational terms (β) dominate, generating genuine qualitative transformation through dimensional expansion.

The ratio α/β thus signals synthesis quality: high α/β ratios indicate surface-level integration (ego-driven), while low α/β ratios indicate deep transformation (archetypal-driven).

3. Transformation Dynamics and Syntony Conditions

The bi-tetradic formalism defines vector operations suitable for modelling psychological and sociological effects.

Figure 1 shows two fundamental transformation sequences that remain after imposing dialectical constraints:

conceptual oppositions must occupy diagonal positions (dashed arrows). In other words, Truth cannot be adjacent to Falsehood, and Thesis cannot be adjacent to Antithesis.

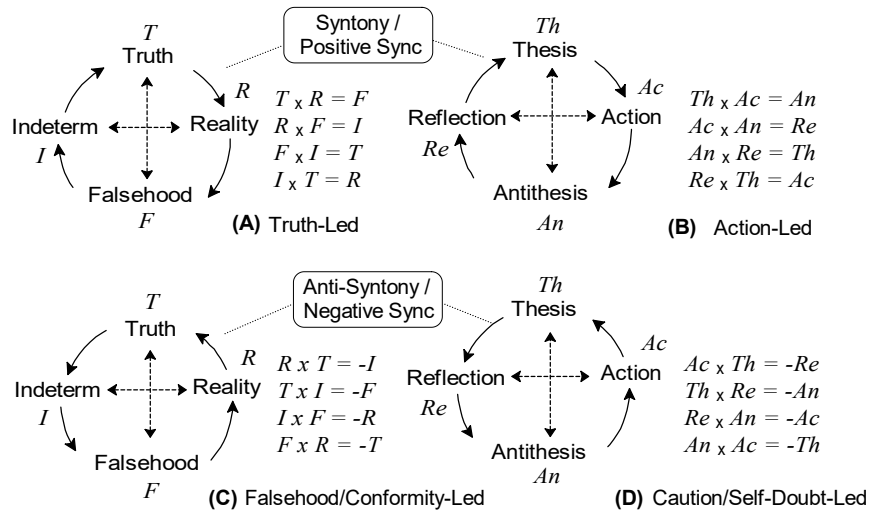


Fig. 1. Synchronized transformations.

Here ‘-’ marks belonging to the reversed cycle, suggesting a quality/valence flip (constructivity vs. destructivity) rather than a change of vector’s direction. (An open question is how to notate this in operations. It seems that the sign depends only on the order of multiplied components rather their respective signs, e.g., $(\pm R) \times (\pm T) \equiv -I$, or $R^{\pm} \times T^{\pm} \equiv I^{-}$, which mathematically may look awkward. GPT suggests $\forall \sigma, \tau \in \{\pm\}: R^{\wedge \sigma} \times T^{\wedge \tau} = I^{\wedge -}$).

Schemes A and B show transformations of Truth or Thesis into Reality/Action, which then produce Falsehood/Antithesis as creative destruction or surprise, and returns as Indeterminacy/Reflection transformed into new Truth/Thesis. This pattern resembles *pragmatic idealists* who shape reality through conviction and practice (since Truth/Thesis determines Reality/Action).

Schemes C and D show the reversed orientation, where Truth/Thesis breeds Indeterminacy/Reflection, then Falsehood/Antithesis, and only then Reality/Action—as a consequence rather than a cause. This pattern can be assigned to *naive materialists*, yielding over-analysis and defensive crystallization that closes off growth (since Reality/Action comes from Falsehood/Antithesis).

The latter parallel is striking, as in many ways it mirrors modern intellectual life, where Truth produces Indeterminacy rather than Reality, and Thesis produces Reflection rather than Action. This resonates with ancient wisdom (cf. Lao Tzu: “Those who talk do not know; those who know do not talk”) and with modern insight (cf. Alan Watts: “Omnipotence is not knowing how everything is done; it is simply doing it”).

Fig. 2 summarises the possible combinations of neutrosophic and dialectic cycles in Fig. 1. The shaded diagonal cells correspond to the two coherent orientations (A, B and C, D), while the off-diagonal cells show “misaligned” cases: the B–C combination (“distorted extraversion”) can devolve into impulsive rebellion typical of inexperienced youth; the A–D combination (“distorted introversion”) may lapse into sterile speculation typical of exhausted elders. The developmental aim is to restore synchronisation of schemes A and B, where Reality is enhanced by simple praxis (cf. Lao Tzu). The dashed arrow marks a common misconception: that maturation must entail de-idealisation, whereas maturity may instead integrate ideal values with experienced insight.

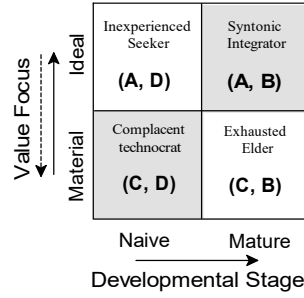


Fig. 2. Possible combinations of cycles in Fig. 1

4. Non-Commutativity and the Arrow of Experience

As we saw above, the coupled system exhibits quaternion-like multiplication where order determines outcome:

$$\mathbf{T} \times \mathbf{I} = \mathbf{F} \neq \mathbf{I} \times \mathbf{T} = \mathbf{R}$$

This non-commutativity captures the "arrow of experience"—Truth engaging Indeterminacy yields Falsehood (premature closure), while Indeterminacy engaging Truth yields Reality (authentic opening). This asymmetry captures the irreversible "arrow of experience." Once a cognitive sequence has been enacted, no simple inverse operation returns the prior state. Unlike thermodynamic processes, psychological transformations exhibit path dependence where the sequence of operations determines the qualitative outcome.

The cross-layer terms also manifest non-associativity, *e.g.*: $(\mathbf{R} \times \mathbf{T}) \times \mathbf{Ac} \neq \mathbf{R} \times (\mathbf{T} \times \mathbf{Ac})$

Under our current interpretive mapping, the first aligns with *Experimentation* (since $R \times T = I$, and $I \times \mathbf{Ac} \approx \text{Experimentation}$), while the second tends to *Fatalism* (if one treats $T \times \mathbf{Ac} \approx \text{Reformation}$, and $R \times \text{Reformation} \approx R \times \mathbf{Ac} = \text{Fatalism}$).

This explains why "unlearning" is practically impossible—consciousness cannot simply reverse its developmental trajectory. Instead, growth requires transcendent integration that encompasses prior stages while opening new possibilities.

The framework thus provides mathematical grounding for why syntony cultivation, rather than naive reversal, becomes the optimal strategy for navigating complex environments.

5. Cross-Layer Combinations

Building on the two orientations described above, the bi-tetradic coupling yields a full 4×4 matrix of 16 cross-layer combinations, each representing a distinct cognitive–affective state (Table 2). Although these assignments were initially generated with AI and are therefore speculative, they highlight resonances between personality theory and systemic semantics and point to potential avenues for empirical validation.

Table 2. Cross-layer combinations and personality correspondences

	Thesis (Th)	Action (Ac)	Antithesis (An)	Reflection (Re)	Pattern
Truth (T)	Self-evident Axiom, Logician	Honest Expression, Reformer	Constructive Opposition, Skeptic	Authentic Integration, Philosopher	Thinking-oriented, systematic
Reality (R)	Cosmic Dogma, Mysticism	Fatalism, Consequentialism	Critical Existentialism	Mystical Insight, Sage	Intuitive, transcendent
Falsehood (F)	Delusion, Ideologue	Manipulation, Machiavellian	Opposition/ Surprise, Nihilist/Wonder	Rationalization, Apologist	Experienced, pragmatic
Indeterminacy (I)	Ambiguous Promise, Explorer	Experiment, Innovation	Paradox, Trickster	Creative Reinterpretation, Poetry or Sophistry	Adaptive, innovative
Pattern	Foundational	Pragmatic	Critical	Integrative	

Each cell can manifest in a scalar-dominant or a rotational-dominant form. For example, $F \times An$ may appear as “Opposition” when scalar terms predominate, or as “Surprise” when rotational terms predominate. $T \times Th$ may range from a quiet “angel’s whisper” to a shout of desperation. Table 2 lists only the most commonly observed interpretations.

Four combinations show strong internal complementarity between neutrosophic and dialectical aspects, forming the “syntonic diagonal”:

1. $T \times Th$ (**Self-Evidence**): Truth grounded in authentic affirmation.

2. **R × Ac (Imminence):** Reality manifesting through direct enactment.
3. **F × An (Surprise/Opposition):** Wonder at unexpected emergence.
4. **I × Re (Poetry/Sophistry):** Creative integration transforming surprise into new understanding.

When these states occur in the sequence shown in Fig. 1 (A, B) they produce syntononic synchronisation—a silent, self-organising flow (Self-Evidence → Imminence → Surprise → Poetry). The reversed sequence (Sophistry → Opposition → Imminence → Evidence) produces anti-syntony, a rigid, scalar-driven force. Breaking this rigidity requires coordinated action of the remaining (non-diagonal) states, which then act as “specialist roles” summarised in the closing row and column. For example, T-types excel in clarity of thinking, R-types in depth, F-types in practicality and wisdom, and I-types in resilience and creativity.

The key point is that syntony cannot be produced by any single state alone but only by the right sequence of interlocking capacities—likely more intricate than our current model can yet describe.

6. Conclusions

This work reframes neutrosophy as a living calculus of transformation rather than a static classification. Its dual-layer architecture links mathematical structure with psychological process, opening a path for empirical testing and designing practices that cultivate reality-enhancing, rather than reality-blocking, states.

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